

No Λ oscillations

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ABSTRACT: We examine a recently published calculation which predicts an oscillatory behaviour for the decay of Λ s produced together with a neutral kaon, and proposes a new expression for the wavelength of kaon strangeness oscillations. We modify the calculation by imposing the requirement that the interference of the K_L and K_S components of the kaon wave function occurs at a specific space-time point. With this requirement, the unusual results predicted vanish, and the conventional results are recovered.

When neutral kaons are produced in a hadronic reaction, strangeness conservation dictates that the kaons are produced in one of the strangeness eigenstates, either a K^0 or a \bar{K}^0 . For example, the reactions

$$\pi^- p \rightarrow \Lambda K^0$$

or

$$K^- p \rightarrow \bar{K}^0 n$$

produce essentially pure K^0 and \bar{K}^0 states respectively. These are mixtures of the well-known mass eigenstates K_L and K_S , e.g.

$$|K^0\rangle = \sqrt{\frac{1+|\epsilon|^2}{2(1+\epsilon)^2}} (|K_S\rangle + |K_L\rangle) \quad (1)$$

where ϵ is the usual CP-violation parameter. Because the K_L and K_S have different lifetimes (and therefore amplitudes) and masses (and therefore phases), the system does not remain in a K^0 state, but oscillates between a K^0 and a \bar{K}^0 , approaching the equal mixture of a pure K_L state. This is the well-known phenomenon of strangeness oscillations[1].

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The dynamics of the $K^0 - \bar{K}^0$ system are nearly always treated in isolation, without regard for other particles in the process. However, in a recent paper, Srivastava, Widom and Sassaroli[2] (denoted by SWS in the following) examined the kinematics of the reaction $\pi^- p \rightarrow \Lambda K^0$. They pointed out that since the K_L and K_S differ in mass by $\delta m = m_{K_L} - m_{K_S} = 3.522 \times 10^{-12}$ MeV, the final state contains a mixture of two momenta for both the kaon and the Λ . In the overall center of mass, the final state is

$$| \Lambda K(t=0) \rangle = \sqrt{\frac{1 + |\epsilon|^2}{2(1 + \epsilon)^2}} \{ | \Lambda_S K_S \rangle + | \Lambda_L K_L \rangle \} \quad (2)$$

at the moment of production, $t = 0$. Denoting the total center-of-mass energy by \sqrt{s} , the center-of-mass momenta of the K_L and K_S are given by

$$p_i = \frac{(s - m_i^2 - m_\Lambda^2)^2 - 4m_i^2 m_\Lambda^2}{4s}$$

where $i = L$ or S . Λ_L and Λ_S denote a Λ in a momentum eigenstate with momentum equal in magnitude to that of the corresponding kaon. We use the subscripts L and S to denote kinematic quantities relevant to the K_L and K_S , quantities without a subscript to the average of these, and the subscript Λ for quantities relevant to the Λ . A consequence of these two momenta is that there are four rest frames relevant to the problem, i.e. those for the K_L , the K_S , the Λ_L and the Λ_S . The proper times in these frames are denoted τ_L , τ_S , τ_{Λ_L} and τ_{Λ_S} respectively.

The state (2) develops in time according to

$$| \Lambda K(t) \rangle = \sqrt{\frac{1 + |\epsilon|^2}{2(1 + \epsilon)^2}} \{ a_S(\tau_{\Lambda_S}, \tau_S) | \Lambda_S K_S \rangle + a_L(\tau_{\Lambda_L}, \tau_L) | \Lambda_L K_L \rangle \} \quad (3)$$

where

$$a_i(\tau_{\Lambda_i}, \tau_i) = \exp\{-i(m_i \tau_i + m_\Lambda \tau_{\Lambda_i}) - \frac{1}{2}(\Gamma_i \tau_i + \Gamma_{\Lambda_i} \tau_{\Lambda_i})\} \quad (4)$$

with $i = S$ or L . The four proper times, τ_L , τ_S , τ_{Λ_L} and τ_{Λ_S} , are related to the time in the overall center-of-mass frame, t , by the appropriate Lorentz transformations relating a point (ξ_i, τ_i) in the frame i to the point (x, t) in the overall center-of-mass frame:

$$\begin{aligned} \xi_i &= \gamma_i(x - \beta_i t) \\ \tau_i &= \gamma_i(t - \beta_i x) \end{aligned}$$

and this transformation must be chosen carefully.

SWS seem to be the first to examine the relation between these proper times. They used a prescription described in an earlier paper[3]. In their work, the form of the K^0 strangeness

oscillations is different from that given by the usual treatment and their results showed several unconventional features, in particular:

(S1) SWS derive a different relation between the wavelength of the strangeness oscillations and the $K_L - K_S$ mass difference, δm . Therefore, they deduce that existing measurements of δm from strangeness oscillations are in error by a factor $C(s)$ which is at least 2 and is much larger near threshold.

(S2) The joint probability distribution, $P(x_\Lambda, x_{K^0}) = |\Psi(x_\Lambda, x_{K^0})|^2$ for detecting both a Λ and a K^0 from the reaction $\pi^- p \rightarrow \Lambda K^0$ will show oscillations for both the K^0 and Λ distributions as a function of distance from the reaction point. For the K^0 , these are the familiar strangeness oscillations, but those for the Λ are a new effect.

These novel features are a consequence of their choice of proper times and of their treatment of two distinct momenta in both the Λ and kaon states.

Our interest in the work of SWS was stimulated initially by the possibility of a direct experimental test of some of their predictions. In the process of investigating this, we re-examined their derivation. While this re-derivation confirmed some of their results, we found some important differences, which changed the conclusions listed above. This letter describes our results.

The essence of the differences between our treatment and that of SWS lies in the relation between the four proper times in the respective rest frames. In deriving these, our starting point is the point at which the experimental observation is made. For either particle, say the kaon, we choose a specific space point x in the overall center-of-mass frame. In the usual plane-wave treatment, the choice of an observation time t is immaterial, since the wave function is present for all time; we are free to observe at any time. However, in any scattering problem, it is implicitly assumed that a more realistic description can be obtained from the usual plane-wave treatment by constructing wave packets. In view of this, the time of observation should be chosen such that the wave packet (or, equivalently, the classical particle) is present at the point x . In the present case, the outgoing kaon is a superposition of two wave packets with different momenta, so would separate after a sufficiently long time. However, this is not an issue here, since the difference in velocity of the K_S and K_L wave packets is small enough ($\sim 10^{-15}$) that they do not separate significantly before detection; it is easy to choose the size of the wave packets to be such that they are large compared with the separation of their centroids over the time of the experiment while still small compared with the dimensions of the apparatus.

Therefore, we choose the time of observation to be the average of when the wave packets for K_S and K_L (and also the classical particles) arrive at point x . The mean velocity of these wave packets is

$$\bar{\beta} = \frac{1}{2}(\beta_L + \beta_S)$$

so that the point of observation in the center-of-mass frame is

$$(x, t) = \left(x, \frac{x}{\bar{\beta}} \right).$$

The choice of $\bar{\beta}$ is not at all crucial in the following. It could be replaced by, for example, β_L or β_S without changing any conclusions. With the choice $\bar{\beta}$, the proper time in the rest frame of particle i is therefore

$$\tau_i = \gamma_i \left(\frac{x}{\bar{\beta}} - \beta_i x \right) = \gamma_i x \left(\frac{1}{\bar{\beta}} - \beta_i \right). \quad (5)$$

The difference between our calculation and that of SWS can be seen at this point. They relate the proper time τ_i to the space point in the center-of-mass, x , by

$$\tau_i^{SWS} = \frac{m_i}{p_i} x = \frac{1}{\beta_i \gamma_i} x$$

which can be expressed as

$$\tau_i^{SWS} = \gamma_i x \left(\frac{1}{\beta_i} - \beta_i \right)$$

which differs from our result by the change from $1/\bar{\beta}$ to $1/\beta_i$. Therefore, because the velocities of the K_L and K_S components differ slightly, SWS are calculating interference at *two different center-of-mass times*. Our treatment uses the *same center-of-mass* time for the detection of the interfering K_L and K_S components, hence the appearance of the same expression for time ($x/\bar{\beta}$) in the Lorentz transformation for both the K_L and K_S . We require that the interference between the two components is calculated at the *same* time and the *same* space point. In SWS, the *center-of-mass times* in their Lorentz transformation are x/β_L and x/β_S for the two components of the neutral kaon.

It is important to realise that this difference is not the result of a technical error in either calculation, but of a difference in principle in the treatment of the quantum mechanics of the system. We believe that it is an error in principle to calculate the interference between wave functions at different points in space-time.

With our expression (5) for τ_L , τ_S , τ_{Λ_L} and τ_{Λ_S} , we can write the coefficients (4) as

$$a_i(t) = \exp \left[-i \left(m_{\Lambda} \gamma_{\Lambda i} \left(\frac{1}{\bar{\beta}_{\Lambda}} - \beta_{\Lambda i} \right) x_{\Lambda} - m_i \gamma_i \left(\frac{1}{\bar{\beta}} - \beta_i \right) x_K \right) \right. \\ \left. - \frac{1}{2} \left(\Gamma_{\Lambda} \gamma_{\Lambda i} \left(\frac{1}{\bar{\beta}_{\Lambda}} - \beta_{\Lambda i} \right) x_{\Lambda} - \Gamma_i \gamma_i \left(\frac{1}{\bar{\beta}} - \beta_i \right) x_K \right) \right] \quad (6)$$

and the state vector at center-of-mass time t as

$$| \Lambda K(t) \rangle = \sqrt{\frac{1 + |\epsilon|^2}{2(1 + \epsilon)^2}} \{ a_S(t) | \Lambda_S K_S \rangle + a_L(t) | \Lambda_L K_L \rangle \} \quad (7)$$

The interesting predictions result from selecting a specific strangeness for the kaon. If we take the K^0 part of (7), we get

$$\begin{aligned}\Psi_{\Lambda K^0}(x_{K^0}, x_\Lambda) &= \sqrt{\frac{1 + |\epsilon|^2}{2(1 + \epsilon)^2}} \{a_S(t)\langle \Lambda K^0 | \Lambda_S K_S \rangle + a_L(t)\langle \Lambda K^0 | \Lambda_L K_L \rangle\} \\ &= \frac{1}{2}\{a_S(t) + a_L(t)\}\end{aligned}$$

and the \bar{K}^0 part has the opposite sign in the bracket, $\{a_S(t) - a_L(t)\}$. Writing $a_i(t)$ as $a_i(t) = \exp(-ib_i - c_i)$, the joint probability distribution for detection of a K^0 and a Λ is given by

$$\begin{aligned}P(x_\Lambda, x_{K^0}) &= \frac{1}{4} |a_S(t) + a_L(t)|^2 \\ &= \frac{1}{4} \{|a_S(t)|^2 + |a_L(t)|^2 + 2e^{-(c_S + c_L)} \cos(b_L - b_S)\}\end{aligned}$$

The cosine term gives the oscillations. From (6),

$$b_i = m_\Lambda \gamma_{\Lambda i} \left(\frac{1}{\beta_\Lambda} - \beta_{\Lambda i} \right) x_\Lambda - m_i \gamma_i \left(\frac{1}{\beta} - \beta_i \right) x_K.$$

The quantity $(b_L - b_S)$ is most readily evaluated using

$$b_L - b_S = \frac{db}{dm} \delta m$$

together with

$$\begin{aligned}\frac{dp}{dm} &= \frac{-m}{2p} \left[1 + \frac{m_\Lambda^2 - m^2}{s} \right] \\ \frac{dE}{dm} &= -\frac{dE_\Lambda}{dm} = \frac{m}{\sqrt{s}}.\end{aligned}$$

From this, we obtain

$$\frac{db_i}{dm} = \frac{m}{p} x_K$$

and hence the cosine term becomes $\cos(kx_K)$, where

$$k = \frac{m \delta m}{p}. \quad (8)$$

There are two striking features of this result

(L1) In contrast to SWS, there is no dependence on x_Λ in db/dm . Thus the oscillations in the Λ probability distribution, predicted by SWS, are not present. Oscillations exist in the Λ probability only in the sense that if we detect the K^0 and the Λ at the same center-of-mass time,

i.e. $x_K/\beta = x_\Lambda/\beta_\Lambda$, then oscillations will be observed. However, these are just a consequence of the kaon strangeness oscillations. If we choose a fixed x_K (or, alternatively, integrate over all x_K , corresponding to not detecting the K^0) the Λ distribution has its usual exponentially decaying form with no oscillation.

(L2) The K^0 distribution oscillates with wave number $k = m\delta m/p$, which is just the usual expression[1]. This is perhaps surprising, since a new feature has been introduced into the kinematics, i.e. the presence of two momenta and four proper times, pointed out by SWS. Apparently, this does not affect the strangeness oscillations.

Our result differs from (S1) and (S2) of SWS due to a basic theoretical difference in the treatments. It is therefore natural to look for an experimental test, to cast light on the situation. The problem here is that the calculation of SWS is applicable only in a very limited number of cases. However, we can examine the experimental situation which was one of the original motivations for SWS's work; they quote Fujii *et al.*[4], who state that values of δm derived from strangeness oscillation measurements appear to be higher than those from regeneration experiments. The paper of Fujii *et al.* certainly gives this impression. Further, this would be readily explained by the result of SWS, since they find a wavelength for strangeness oscillations that differs, for a given δm , by over a factor of 2 from our result and hence from the conventional result. However, more recent experiments do not confirm the assertion of Fujii *et al.*[4]. For example, Chang *et al.*[5] list eight measurements of δm from strangeness oscillations of which only 2 early measurements are higher than the results from regeneration experiments.

Direct measurements of strangeness oscillations have been made by many groups. For example, Gjesdal *et al.*[6] used K_{e3} decays to determine the K^0 and \bar{K}^0 components of a beam which was initially predominantly \bar{K}^0 . Their measurements agree well with the conventional description, and therefore with our result. In making this comparison, it is important to realise that Gjesdal *et al.* used a value for δm that is consistent with that from regeneration experiments, thus confirming our expression (7) for the wavelength. The applicability of SWS's result to ref. [6] is not clear. However, they state[7] that the experimental conditions in the measurement on $K^-p \rightarrow \bar{K}^0 n$ by Camerini *et al.*[8] are such that their theory should apply. Nevertheless, Camerini *et al.* measure a value for δm from their experiment, $\delta m = (0.50 \pm 0.15) \tau_S^{-1}$, which is again consistent with the regeneration value, $\delta m = 0.476 \tau_S^{-1}$, which would seem to support our result.

Several preprints have appeared recently which treat various aspects of the problem of interest here, i.e. the quantum oscillations of a particle produced in a 2-body final state. For example, Kayser[9] discusses $B\bar{B}$ mixing and, in particular, the Einstein-Podolsky-Rosen aspects. Grimus and Stockinger[10] and Goldman[11] discuss neutrino oscillations from reactor neutrinos and pion decay respectively. Although related to the topic discussed here, none of these papers addresses directly the questions dealt with in the present work. Thus no direct comparison with these papers is possible except to note that none of them draws any conclusion that is in conflict with our results.

In summary, the paper of SWS introduces two new aspects into the treatment of kaon strangeness oscillation, the presence of two momenta in the reaction that produces the kaons and a new treatment of the proper times of the various states. We believe that the former is correct but without substantial effect on the experimental predictions. The latter, which is the source of their novel

predictions, seems to us in error, based on interference between components of a wave function at different space-time points. Our work incorporates just the first of these features. To the extent that an experimental test is possible, their novel results do not seem to be supported by experiment. We hope that a definitive experiment to give a cleaner distinction between the two calculations will be carried out in the near future.

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